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THEORETICAL INVESTIGATION OF CENTRIFUGATION MECHANICS IN THE FREE VOLUME OF A ROTATING FLUID

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The solution of the equations of the motion of particles in the free volume of a rotating fluid relative to inertial and noninertial coordinate systems has been investigated. It has been established that the centrifugation phenomenon is the drift caused by the action of crossed components of two physical forces: buoyancy force and hydrodynamic resistance.

Keywords: *centrifugation, separation, inertial and noninertial frames of reference, disperse particles, dispersion medium, cluster nanostructure of a fluid.*

Introduction. Despite the wide application of centrifugation in chemical technology, the physical principles of the mechanics of separation of disperse particles of different densities in the volume of a rotating liquid dispersion medium have not been studied as yet. According to the definition adopted in [1], centrifugation is the "separation of suspensions, emulsions, and three-component systems under the action of centrifugal forces." A theoretical description of this process is usually based on linearized equations of the motion of particles relative to a rotating (noninertial) frame of reference. In linearization the Coriolis inertial force and so-called inertial term of the equation, i.e, the time derivative of the pulse of particles, are "neglected" [2–7]. This results in the zero sum of the centrifugal inertial force and of two physical (Newtonian [8]) forces: the horizontal component of the buoyancy force owing its origin to the parabolic pressure gradient inside a rotating dispersion medium and the resistance force that depends on the relative velocity of disperse particles. As is known, with the principal vector equal to zero, the particles should be at rest or move uniformly and rectilinearly; however, in such an approximation, the equation obtained for the velocity of the particles undergoing separation depends on the distance to the axis around which the dispersion medium (fluid) rotates. Consequently, the method of linearization (the Stokes approximation) used in the existing theory of centrifugation contains a logical controversy. Moreover, the experiment shows (see, e.g., [9]) that usually the trajectories of disperse particles in a rotating fluid have a complex form.

In the theoretical hydrodynamics the principal factor of centrifugation is considered to be the resultant of the sum of the centrifugal inertial force and horizontal component of the buoyancy force. The separation mechanics is explained as follows [10]: "If the density of the bodies rotating together with the fluid is higher than the fluid density, such bodies in the rotating fluid ... will be rejected to the periphery, but if the density of the bodies is smaller than the fluid density, such bodies ... will approach the rotation axis." It is customary to consider such a description canonical, even though it has been established experimentally that there is no separation of highly dispersed particles in the free volume of a rotating fluid despite the difference in the densities of the dispersed and dispersion media. Due to this, ultracentrifuges and separators with rotors filled with plates (trays) that provide the process of so-called thin-layer separation were devised.

We note the main drawback of the generally accepted theory of centrifugation that explains the separation of the dispersed particles by the action of the centrifugal inertial force. Centrifugal apparatuses installed both in industrial buildings and in laboratories are located in the geocentric frame of reference. As is known, this system of reference connected with the Earth is adopted as the inertial one for solving the majority of practical problems, where it is needed to calculate the motion of machines and mechanisms, investigate technological processes, and so on. It is evident that the motion of the Earth does not influence the process of centrifugation, and from the viewpoint of the per-

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sonnel who observe the rotating rotors of the apparatuses from the geocentric frame of reference, the sedimentation and floatation of separated particles must occur without regard for any inertial forces.

Modeling of the process of centrifugation in inertial and noninertial frames of reference made in [11] with the aid of a numerical experiment has shown that the conditions of separation of particles in the volume of a rotating fluid are determined by the ratio of two physical forces: the resistance and the radial component of the buoyancy force.

In the present work we suggest analytical solutions of the equations of motion of particles placed into a rotating fluid relative to the inertial frame of reference (IFR), as well as the noninertial one (NFR) connected with the rotating medium. Motion in the NFR has been investigated with account for the centrifugal and Coriolis inertial forces. The calculations were performed for the case of motion in a free volume, i.e., only the initial conditions were used in solving the corresponding Cauchy problem. To solve such a problem with account for the boundary conditions needed, for example, to describe the process of a thin-layer separation in liquid separators, it is required to consider specially the physically justified calculation of reactions on the solid surfaces of the trays located in the volume of the rotating medium.

1. Theoretical. *1.1. Analytical solution of the equation of motion in the noninertial (rotating) frame of reference.* We will consider the motion of a material point — the center of masses of the particle placed inside a fluid that rotates stationarily with the angular velocity ω. The Cartesian coordinate system is connected with the rotating medium. The z^* axis is directed vertically upwards and is coincident with the axis of rotation. The x^* and y^* axes are located in the horizontal plane; they rotate with the angular velocity ω, i.e., the fluid is at rest in the given frame of reference. The vector ω has a constant positive projection on the z^* axis.

The general form of the law of the dynamics of relative motion of the point is

$$m_0\mathbf{a}^* = \mathbf{G} + \mathbf{F}_\text{A} + \mathbf{F}_\text{s} + \hat{\mathbf{O}}_1 + \hat{\mathbf{O}}_2 \, . \tag{1}$$

Motion along the vertical axis is not considered, since the corresponding equation has well-known solutions for both the linear and square laws of resistance. The differential equations of point motion in projections on a horizontal plane with account for the linear law of resistance result from (1) in the form

$$\ddot{x}^* = 2\omega\dot{y}^* + (1 - K)\,\omega^2 x^* - \mu\dot{x}^* \, , \quad \ddot{y}^* = -\,2\omega\dot{x}^* + (1 - K)\,\omega^2 y^* - \mu\dot{y}^* \, . \tag{2}$$

Equations (2) are solved (integrated) with the aid of the substitution

$$\xi = x^* + iy^* \, .$$

After multiplication of the second equation in (2) by i and term-by-term summation, we have a linear homogeneous differential equation for the complex variable ξ:

$$\ddot{\xi} + (\mu + 2\omega i)\,\dot{\xi} + (K - 1)\,\omega^2\xi = 0 \, . \tag{3}$$

The general solution of Eq. (3) is

$$\xi = (C_1 + iC_2)\exp(P_1 t) + (C_3 + iC_4)\exp(P_2 t) \, . \tag{4}$$

To calculate the roots of the characteristic equation (4), P_1 and P_2, the following equations were obtained:

$$P_1 = \beta_1 + i\gamma_1 \, ; \quad P_2 = \beta_2 - i\gamma_2 \, . \tag{5}$$

Here

$$\beta_1 = \frac{\mu}{2} + \sqrt{R}\cos\frac{\varphi}{2} \, ; \quad \beta_2 = -\left(\frac{\mu}{2} + \sqrt{R}\cos\frac{\varphi}{2}\right);$$

$$\gamma_1 = \sqrt{R}\sin\frac{\varphi}{2} - \omega\,;\quad \gamma_2 = \sqrt{R}\sin\frac{\varphi}{2} + \omega\,;\quad R = \sqrt{A^2 + B^2}\,;$$

$$\varphi = \arctan(B/A)\,;\quad A = \frac{\mu^2}{4} - \omega^2 K\,;\quad B = \omega\mu\,.$$

Separating the real and imaginary parts in (4) and performing simple transformations, we find the law of motion of the point relative to the NFR with the rotating plane x^*–y^*:

$$x^*(t) = (C_1\cos\gamma_1 t - C_2\sin\gamma_1 t)\exp(\beta_1 t) + (C_3\cos\gamma_2 t + C_4\sin\gamma_2 t)\exp(\beta_2 t)\,,$$

$$y^*(t) = (C_1\sin\gamma_1 t + C_2\cos\gamma_1 t)\exp(\beta_1 t) - (C_3\sin\gamma_2 t - C_4\cos\gamma_2 t)\exp(\beta_2 t)\,. \tag{6}$$

Differentiation in (6) yields expressions for the projections of velocities with account for which we obtain equations for the coefficients

$$C_1 = x_0 - C_3\,,\quad C_2 = y_0 - C_4\,,\quad C_3 = \frac{D_1 D_2 - D_3 D_4}{D_5}\,,\quad C_4 = \frac{D_1 D_4 + D_2 D_3}{D_5}\,,$$
$$D_1 = \dot{x}_0 - \beta_1 x_0 + \gamma_1 y_0\,,\quad D_2 = \beta_2 - \beta_1\,,\quad D_3 = \dot{y}_0 - \beta_1 y_0 - \gamma_1 x_0\,,\quad D_4 = \gamma_1 + \gamma_2\,,\quad D_5 = D_2^2 + D_4^2\,. \tag{7}$$

Equations (6) determine the coordinates of the point and, as is known, are the parametric equations of its trajectory — in the given case, relative to the rotating NFR. The distance from the rotation axis to the point investigated is

$$r^*(t) = \sqrt{(x^*)^2 + (y^*)^2}\,. \tag{8}$$

It is evident that the dependence of the value of r^* on time can be used as an obvious criterion of centrifugation that characterizes the direction and the velocity of particle displacement relative to the rotation axis.

We will consider the variant of the particle motion relative to the rotating medium, i.e., without account for the resistance. At $\mu = 0$ the differential equations (2) have the form

$$\ddot{x}^* = 2\omega\dot{y}^* + (1 - K)\,\omega^2 x^*\,,\quad \ddot{y}^* = -2\omega\dot{x}^* + (1 - K)\,\omega^2 y^*\,. \tag{9}$$

Using the same substitution $\xi = x^* + iy^*$, we obtain

$$\ddot{\xi} + (2\omega i)\,\dot{\xi} + (K - 1)\,\omega^2\xi = 0\,. \tag{10}$$

The general solution of Eq. (10) for the complex variable ξ is written in the former form, Eq. (4), since the corresponding characteristic equation has two different roots:

$$P_1 = -i\omega\beta\,,\quad P_2 = -i\omega\gamma\,,\quad \beta = 1 - \sqrt{K}\,,\quad \gamma = 1 + \sqrt{K}\,. \tag{11}$$

The separation of the real and imaginary parts of the solution of Eq. (10) yields the following expressions for the law of the particle motion:

$$x^*(t) = C_1\cos(\omega\beta t) + C_2\sin(\omega\beta t) + C_3\cos(\omega\gamma t) + C_4\sin(\omega\gamma t)\,,$$
$$y^*(t) = -C_1\sin(\omega\beta t) + C_2\cos(\omega\beta t) - C_3\sin(\omega\gamma t) + C_4\cos(\omega\gamma t)\,, \tag{12}$$

where

$$C_1 = -\frac{\dot{y}_0 + x_0\omega\gamma}{\omega(\beta - \gamma)}; \quad C_2 = \frac{\dot{x}_0 - y_0\omega\gamma}{\omega(\beta - \gamma)}; \quad C_3 = \frac{\dot{y}_0 + x_0\omega\beta}{\omega(\beta - \gamma)}; \quad C_4 = -\frac{\dot{x}_0 - y_0\omega\beta}{\omega(\beta - \gamma)} . \tag{13}$$

1.2. Analytical solution of the equation of motion in the inertial (fixed) frame of reference. In the inertial frame of reference the coordinate axes are fixed. A liquid medium rotates stationarily around the axis that coincides with the z axis directed upwards on which the angular velocity vector has a constant positive projection. The x and y axes are located in the horizontal plane. According to the definitions accepted in classical dynamics, the motion of an investigated point in a fixed coordinate system is called absolute; the rotating medium performs migratory motion.

The relative velocity vector $\mathbf{V}_{\text{rel}}$ of the investigated point (relative to the rotating medium) is

$$\mathbf{V}_{\text{rel}} = \mathbf{V} - [\boldsymbol{\omega} \times \mathbf{r}] . \tag{14}$$

According to the 2nd law of Newtonian dynamics, during the motion of the point in the IFR, only the following physical forces are in operation:

$$m_0\mathbf{a} = \mathbf{G} + \mathbf{F}_{\text{a}} + \mathbf{F}_{\text{s}} . \tag{15}$$

Not considering the motion along the vertical, we will first investigate the case where only the horizontal component of the buoyancy force is applied to the point. The law of dynamics is written as

$$m_0\mathbf{a}_r = \mathbf{F}_{\text{A}r} , \quad \mathbf{F}_{\text{A}r} = -m\omega^2\mathbf{r} . \tag{16}$$

The differential equations of motion on the horizontal plane are obtained from Eq. (16) in the form

$$\ddot{x} + K\omega^2 x = 0 , \quad \ddot{y} + K\omega^2 y = 0 . \tag{17}$$

Equations of the type of Eq. (17) are well known in the theory of periodic processes. Their general solution can be found by a standard method using the characteristic equation

$$x(t) = C_1 \sin k_0 t + C_2 \cos k_0 t , \quad y(t) = C_3 \sin k_0 t + C_4 \cos k_0 t , \tag{18}$$

where $k_0 = \omega\sqrt{K}$; $C_1 = \dot{x}_0/k_0$; $C_2 = x_0$; $C_3 = \dot{y}_0/k$; $C_4 = y_0$. In vibration theory, Eqs. (18) are usually given in the form

$$x(t) = X\cos(k_0 t + \alpha_1) , \quad y(t) = Y\cos(k_0 t + \alpha_2) . \tag{19}$$

Note that the periodic process described by Eqs. (18) or (19) here presents a periodic change in the coordinates of the point investigated. In this case the point moves along the closed trajectory. With account for the equality of the frequencies on two orthogonal axes, from Eqs. (19) we obtain, with the aid of simple manipulations, the well-known equation of the trajectory in the canonical form:

$$\left(\frac{x}{X}\right)^2 + \left(\frac{y}{Y}\right)^2 - \frac{2xy}{XY}\cos(\alpha_2 - \alpha_1) = \sin^2(\alpha_2 - \alpha_1) . \tag{20}$$

From Eq. (20) we find that

1) at the difference of the initial phases $(\alpha_2 - \alpha_1) = 0, \pi$ the point moves along a segment of a straight line:

$$x/y = X/Y ; \quad x/y = -X/Y ;$$

2) at $(\alpha_2 - \alpha_1) = \pi/2$ and $X = Y = L$ the equation of the circle is obtained:

$$x^2 + y^2 = L^2 ;$$

3) the condition $(\alpha_2 - \alpha_1) = \pi/2$; $X \neq Y$ yields the equation of the reduced ellipse:

$$(x/X)^2 + (y/Y)^2 = 1\ ;$$

4) all the remaining conditions determine the trajectories of the type of unreduced ellipses.

Below the forms of the trajectories in a rotating coordinate system that correspond to the conditions of finite motion of the point in the inertial frame of reference are shown.

We will complete the analytical investigation by solving the equations of motion of the point in the IFR with account for the horizontal component of the linear resistance and buoyancy force. Using Eq. (14) for the relative velocity of the point, we obtain the following differential equations of motion on a fixed horizontal plane:

$$\ddot{x} = -K\omega^2 x - \mu\omega y - \mu\dot{x}\ , \quad \ddot{y} = -K\omega^2 y + \mu\omega x - \mu\dot{y}\ . \tag{21}$$

The solution of Eqs. (21) is obtained by the method considered in Sec. 1.1, i.e., with the use of the substitution $\xi = x + iy$. The differential equation for the complex variable ξ is obtained in the form

$$\ddot{\xi} + \mu\dot{\xi} + (\omega^2 K - \omega\mu i)\,\xi = 0\ . \tag{22}$$

The corresponding characteristic equation has two different roots:

$$P_1 = \beta_1 + i\gamma_1\ , \quad P_2 = \beta_2 + i\gamma_2\ , \tag{23}$$

where

$$\beta_1 = -\frac{\mu}{2} + \sqrt{R}\cos\frac{\varphi}{2}\ ; \quad \beta_2 = -\left(\frac{\mu}{2} + \sqrt{R}\cos\frac{\varphi}{2}\right); \quad \gamma_1 = \sqrt{R}\sin\frac{\varphi}{2}\ ;$$

$$\gamma_2 = -\sqrt{R}\sin\frac{\varphi}{2}\ ; \quad R = \sqrt{A^2 + B^2}\ ; \quad \varphi = \arctan(B/A)\ ; \quad A = \frac{\mu^2}{4} - \omega^2 K\ ; \quad B = \omega\mu\ .$$

Having separated the real and imaginary parts of the general solution for ξ (see Eq. (4)), we obtain the law of motion of the point on the fixed plane x–y:

$$\begin{aligned} x(t) &= (C_1 \cos\gamma_1 t - C_2 \sin\gamma_1 t)\exp(\beta_1 t) + (C_3 \cos\gamma_2 t - C_4 \sin\gamma_2 t)\exp(\beta_2 t)\ , \\ y(t) &= (C_1 \sin\gamma_1 t + C_2 \cos\gamma_1 t)\exp(\beta_1 t) + (C_3 \sin\gamma_2 t + C_4 \cos\gamma_2 t)\exp(\beta_2 t)\ , \end{aligned} \tag{24}$$

where

$$\begin{gathered} C_1 = x_0 - C_3\ ; \quad C_2 = y_0 - C_4\ ; \quad C_3 = \frac{D_1 D_2 + D_3 D_4}{D_5}\ ; \quad C_4 = \frac{D_2 D_3 - D_1 D_4}{D_5}\ ; \\ D_1 = \dot{x}_0 - \beta_1 x_0 + \gamma_1 y_0\ ; \quad D_2 = \beta_2 - \beta_1\ ; \quad D_3 = \dot{y}_0 - \beta_1 y_0 - \gamma_1 x_0\ ; \quad D_4 = \gamma_2 - \gamma_1\ ; \quad D_5 = D_2^2 + D_4^2\ . \end{gathered} \tag{25}$$

It should be emphasized that the expressions for the laws of motion, Eqs. (24) and (18), just as Eqs. (12) and (6), are not identical at $\mu = 0$. From the mathematical point of view, this is attributable to the different forms of the roots of the corresponding characteristic equations. As is shown below, investigation of the trajectories of centrifugated particles allows one to obtain a fuller notion on the physical mechanism of separation in a rotating fluid.

2. Investigation of Trajectories and Discussion of Results. *2.1. Techniques.* Analytical solutions of the Cauchy problem, which were presented in Sec. 1, allow one to investigate the trajectories of particles depending on time as a continuous variable, when the parameters ω, K, and μ, as well as the initial conditions are varied. The pa-

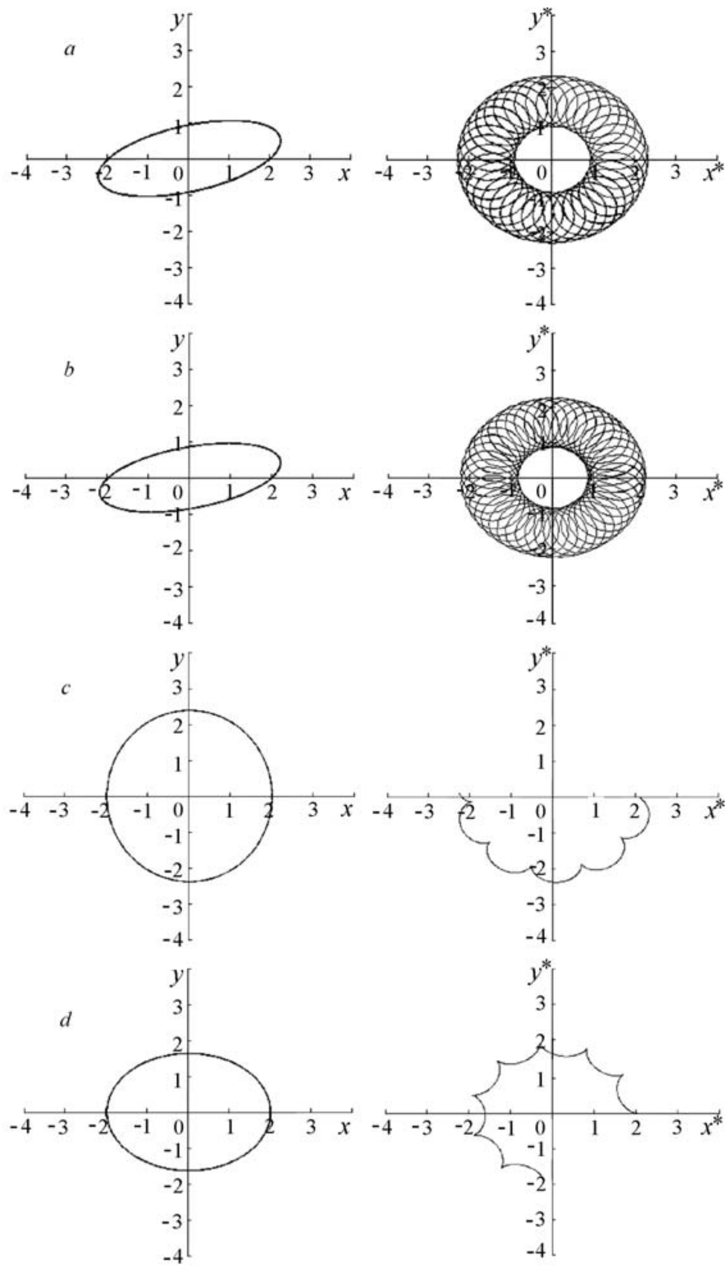

Fig. 1. Trajectories of particles without account for the resistance (IFR on the left; NFR on the right): a) $\omega = 2$; $x_0 = 2$, $y_0 = 0$, $K = 0.9$, $V_{0x} = 2$, $V_{0y} = 2$; b) $\omega = 2$, $x_0 = 2$, $y_0 = 0$, $K = 1.1$, $V_{0x} = 2$, $V_{0y} = 2$; c) $\omega = 5$, $x_0 = 2$, $y_0 = 0$, $K = 0.7$, $V_{0x} = 0$, $V_{0y} = 10$; d) $\omega = 5$, $x_0 = 2$, $y_0 = 0$, $K = 1.5$, $V_{0x} = 0$, and $V_{0y} = 10$. x, y, m.

rameters ω and μ have the dimensionality sec^{-1}, the linear dimensions can be multiplies of the length unit in the SI system.

The investigations have been carried out with the use of the computer program MathCAD. The trajectories of particles were observed on the monitor screen simultaneously in two frames of reference: inertial and noninertial. The graphical representations were the "view from above" onto horizontal planes with the axes (x–y) and (x^*–y^*). At the initial instant of time the coordinate axes of the two frames of reference were superposed; for the coordinates at $t = 0$ the following constant values were given: $x_0 = x_0^* = 2$ and $y_0 = y_0^* = 0$. The initial velocities changed in a wide range,

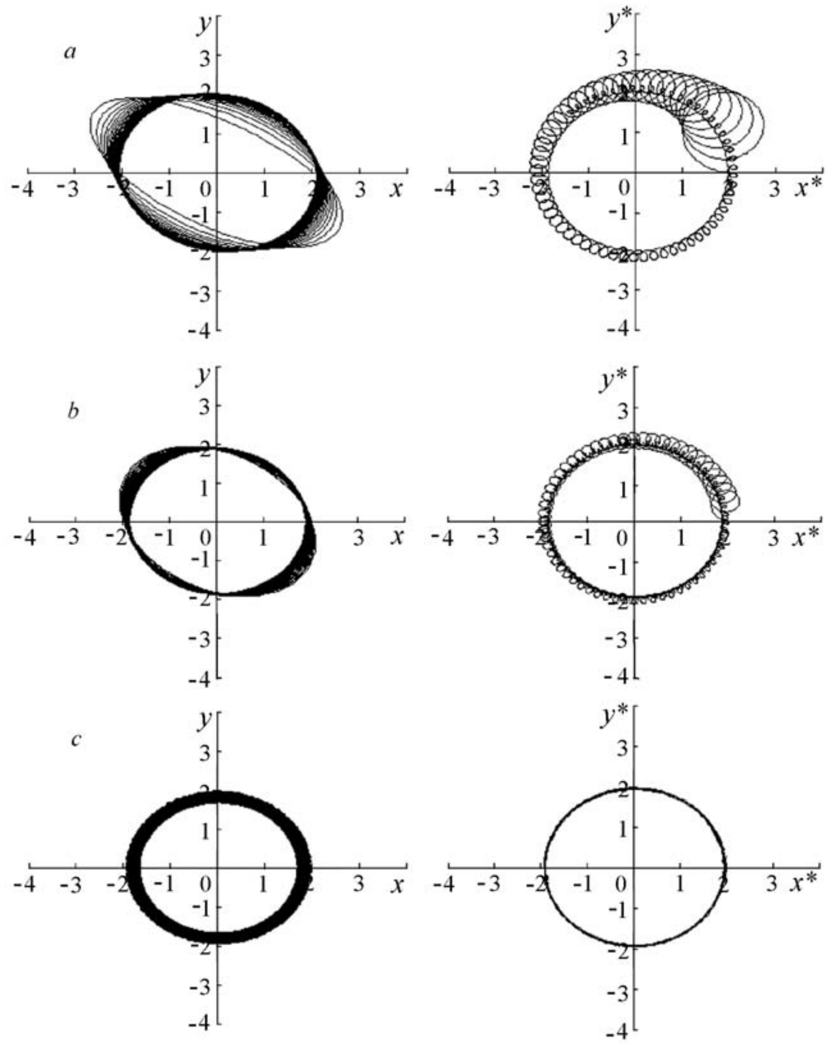

Fig. 2. Trajectories of particles with account for small resistance (IFR on the left; NFR on the right): a) $\mu = 0.025$, $\omega = 1$, $x_0 = 2$, $y_0 = 0$, $K = 1.0695$, $V_{0x} = -2$, $V_{0y} = 0$; b) $\mu = 0.025$, $\omega = 3$, $x_0 = 2$, $y_0 = 0$, $K = 1.0695$, $V_{0x} = -2$, $V_{0y} = 0$; c) $\mu = 0.025$, $\omega = 50$, $x_0 = 2$, $y_0 = 0$, $K = 1.0695$, $V_{0x} = -2$, $V_{0y} = 0$. x, y, m.

and it was taken into account that $V_{0x} = V_{0x^*}$ and $V_{0y^*} = V_{y0} - \omega x_0$. The parameters μ, ω, and K have the same values for the fixed and rotating frames of reference.

The correspondence of the trajectories obtained by plotting the functions $x = x(t)$, $y = y(t)$ and $x^* = x^*(t)$, $y^* = y^*(t)$ was checked with the acid of the well-known formulas for converting coordinates from a rotating frame of reference into a fixed one and conversely.

2.2. Discussion of results. Figures 1–3 demonstrate a number of characteristic trajectories of particles in different systems of reference. On the left there are trajectories in IFR relative to which the liquid rotates in an anticlockwise direction; the particles move in the fixed plane (x–y). On the right there are NFR, where the liquid is at rest; the particles move in the plane (x^*–y^*), which rotates together with the liquid medium.

The trajectories at $\mu = 0$ (Fig. 1) illustrate the inference made in Sec. 1.2 that under the action of the buoyancy force $\mathbf{F}_{\mathrm{A}r}$ any particles, irrespective of the values of the parameters ω and K, must move in IFR along the finite trajectories of the type of ellipses. Note that this result follows also from the general laws of the mechanics of motion in the field of the central conservative forces to whose type the $\mathbf{F}_{Ar}$ force relates. The field of the $\mathbf{F}_{Ar}$ force has a specific property: the potential increases proportionally to the squared distance from the rotation axis, and this provides

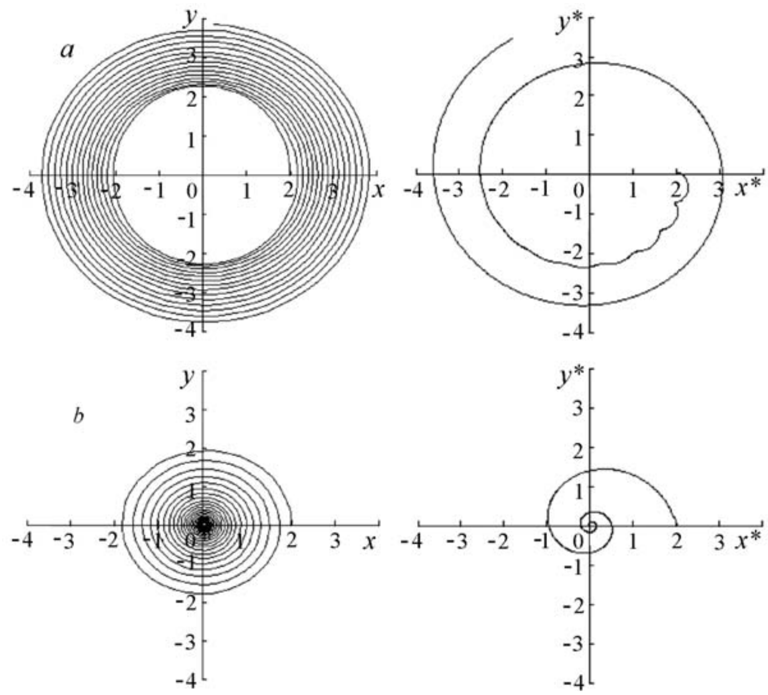

Fig. 3. Trajectories of particles with account for resistance (IFR on the left; NFR on the right): a) $\mu = 5$, $\omega = 50$, $x_0 = 2$, $y_0 = 0$, $K = 0.8$, $V_{0x} = 0$, $V_{0y} = 2$; b) $\mu = 5$, $\omega = 10$, $x_0 = 2$, $y_0 = 0$, $K = 1.25$, $V_{0x} = 0$, $V_{0y} = 2$. x, y, m.

the condition of the closeness of orbits at any initial conditions. From the physical point of view, the indicated property of the buoyancy force field is attributed to the quadratic dependence of the concentration of structural elements, i.e, clusters of the rotating liquid, due to which the parabolic pressure gradient originates [12, 13].

The trajectories shown in Fig. 1 have different forms in the fixed and rotating frames of reference. In the NFR particles move along the trajectories of the type of epicycloids, with "heavy" particles ($K < 1$) and "light" ones ($K > 1$) having opposite directions of displacements about the axis. To demonstrate such displacements, Fig. 1 presents the initial sections of trajectories in NFR; the trajectories become closed with time.

As is shown in Figs. 2 and 3, account for the resistance changes the form of the trajectories of particles and determines the separation conditions. Investigations have shown that under the condition of a small difference between the densities $\Delta\rho = \rho - \rho_0$ and at small coefficients of resistance the particles move on the average at a constant distance from the rotation axis. The increase in the angular velocity does not provide the separation of such particles, transforming only the trajectories to the forms of circles in both systems of reference (Fig. 2). This result agrees with the practically known absence of deposition of highly dispersed particles, which is usually attributable to the diffusion pressure gradient that balances out the "separation force." The actual reason for the absence of separation is that the resistance becomes small when the dimensions of the highly dispersed particles become comparable with the dimensions of the clusters of the liquid dispersion medium. In this case, the influence of the pressure gradient on the surface of separated particles disappears, i.e., they become part of the dispersion phase.

Reliable separation, i.e., flotation of "light" particles and sedimentation of "heavy" ones, is provided only at a large enough resistance. This process is also improved with increase in $\Delta\rho$. The separated particles (see Fig. 3) move along spiral trajectories (excluding the initial section, which may have a "loop" form depending on the parameters).

In the process of the computer experiment, dozens of variants of the motion of particles at $\mu \neq 0$ for different initial velocities and parameters ω, K, and μ were investigated. The time dependences of the distances from the rotation axis and velocities of particles were studied. Integration of the differential equations of the motion of particles was performed also by the Runge–Kutta method with account for both the linear and quadratic laws of resistance. The quadratic dependence of resistance on velocity practically does not change the form of the trajectories obtained for the linear resistance at the same parameters and initial conditions.

The investigations carried out confirm the inference made in [11] that centrifugation in the free volume of a rotating fluid is provided by the action of two physical forces: the centrifugal component of the buoyancy force and the resistance appearing in motion of particles relative to the liquid medium. Both forces are a measure of the real factor of interaction between a particle and clusters of the rotating liquid medium. In this case, "heavy" particles are shifted to the periphery, and "light" ones to the rotation axis. This displacement is a kind of drift in the process of which both types of particles move relative to the rotating fluid along the spiral trajectories in the opposite directions.

Conclusions. The numerical experiment carried out on the basis of the exact solutions of differential equations of motion of disperse particles in the volume of a rotating fluid has shown that to understand the mechanics of centrifugation of different-density particles, account for the most novel data on the nanostructure of the liquid state of substance is needed.

Of interest is the investigation of a more complex model of disperse particles when the system of external forces becomes equivalent not to the resultant but rather to two crossed forces. Consequently, the moment of a pair of forces appears that causes the rotation of particles. In this case the forms of a trajectories of mass centers can change. Such an investigation is useful also for the development of a new approach (considered in [14]) to the modeling of vortex structures observed in registration of the trajectories of flares in experiments with rotating basins.

NOTATION

$\mathbf{a}$, absolute acceleration of the point; $\mathbf{a}_r$, radial acceleration of the point in IFR; $\mathbf{a}^*$, vector of relative acceleration of the point; A and B, auxiliary variables; C_1–C_4, constants determined by initial conditions; D_1–D_5, auxiliary variables; $\mathbf{F}_\mathrm{A}$, Archimedes (buoyancy) force including vertical and horizontal components; $\mathbf{F}_{\mathrm{A}r}$, horizontal component of Archimedes force; $\mathbf{F}_\mathrm{s}$, resistance for which a linear dependence on the velocity of the point relative to the fluid is adopted; $\mathbf{G}$, gravity force; $i = \sqrt{-1}$, imaginary unit; $K = \rho/\rho_0$, auxiliary variable; k_0, intrinsic cyclic frequency of vibrations; m_0, mass of the point; P_1 and P_2, roots of characteristic equation; $\hat{\mathbf{Q}}_1$ and $\hat{\mathbf{Q}}_2$, centrifugal and Coriolis inertia forces; $\mathbf{r}$, radius-vector of the point; $r^*(t)$, distance from the rotation axis to the point studied; R, auxiliary variable; t, time; $\mathbf{V}$, vector of absolute velocity of the moving point studied; $\mathbf{V}_\mathrm{rel}$, vector of relative velocity; V_{0x} and V_{0y}, initial velocities along the coordinate axes; X and Y, constants determining the amplitude of vibrations; x, y, z, coordinate axes in the inertial (fixed) coordinate system; x^*, y^*, z^*, coordinate axes in the noninertial (rotating) coordinate system; x_0, x_0^*, y_0, and y_0^*, initial coordinates of the point in two frames of reference at the initial instant of time $t = 0$; α_1 and α_2, constants determining the initial phase of vibrations; β_1, β_2, γ_1, and γ_2, auxiliary variables; μ, constant specific coefficient of resistance; ρ, density of the mass of the liquid displaced by the volume of the particle; ρ_0, density of the particle mass; φ, auxiliary variable; $\boldsymbol{\omega}$, angular velocity; $[\boldsymbol{\omega}\times\mathbf{r}]$, vector of the velocity of following. Subscripts: A, Archimedes force; r, from radius-vector; s, resistance; 0, initial value; asterisk, indicates that the quantity belongs to the noninertial (rotating) coordinate system; dots over variables, derivative with respect to the parameter t.

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